

Inhomogeneous Fulde-Ferrell superfluidity in spin-orbit coupled atomic Fermi gases

Xia-Ji Liu^{1*} and Hui Hu¹

¹*ARC Centres of Excellence for Quantum-Atom Optics and Centre for Atom Optics and Ultrafast Spectroscopy, Swinburne University of Technology, Melbourne 3122, Australia*

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Inhomogeneous superfluidity lies at the heart of many intriguing phenomena in quantum physics. It is believed to play a central role in unconventional organic or heavy-fermion superconductors, chiral quark matter, and neutron star glitches. However, so far even the simplest form of inhomogeneous superfluidity, the Fulde-Ferrell (FF) pairing state with a single centre-of-mass momentum, is not conclusively observed due to the intrinsic complexity of any realistic Fermi systems in nature. Here we theoretically predict that the controlled setting of ultracold fermionic atoms with synthetic spin-orbit coupling induced by a two-photon Raman process, demonstrated recently in cold-atom laboratories, provides a promising route to realize the long-sought FF superfluidity. At experimentally accessible low temperatures (i.e., $0.05T_F$, where T_F is the Fermi temperature), the FF superfluid state dominates the phase diagram, in sharp contrast to the conventional case without spin-orbit coupling. We show that the finite centre-of-mass momentum carried by Cooper pairs is directly measurable via momentum-resolved radio-frequency spectroscopy. Our work opens the way to direct observation and characterization of inhomogeneous superfluidity.

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Interacting Fermi systems with imbalanced spin populations are quite ubiquitous in nature, with manifestations ranging from solid-state superconductors to astrophysical objects [1]. The spin-imbalance disrupts the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity, where fermions of opposite spin and momentum form Cooper pairs. As a result, an exotic superconducting state, characterized by Cooper pairs with a finite centre-of-mass momentum and spatially non-uniform order parameter, may occur, as predicted by Fulde and Ferrell nearly fifty years ago [2]. More complicated inhomogeneous pairing states are also possible with the inclusion of more and more momenta for the spatial structure of order parameter, following the idea by Larkin and Ovchinnikov (LO) [3]. These forms of inhomogeneous superfluidity, now referred to as FFLO states, have attracted tremendous theoretical and experimental efforts over the past five decades [4]. Remarkably, to date there is still no conclusive experimental evidence for FFLO superfluidity. In solid state systems such as the organic superconductor λ -(BETS)₂FeCl₄ [5] and the heavy fermion superconductor CeCoIn₅ [6, 7], the experimental difficulty arises from unavoidable disorder effects and orbit/paramagnetic depairings close to the upper critical Zeeman field.

An ultracold atomic Fermi gas has proven to be an ideal tabletop system for the pursuit of FFLO superfluidity [4]. Although it is largely analogous to an electronic superconductor, the high controllability in interactions, spin-populations and purity leads to a number of unique experimental advances [8]. Indeed, following theoretical proposals by Orso [9] and the present authors [10–12], strong experimental evidence for the FFLO pairing has been observed in a one-dimensional spin-imbalanced Fermi gas of ⁶Li atoms [13]. In three dimensions (3D),

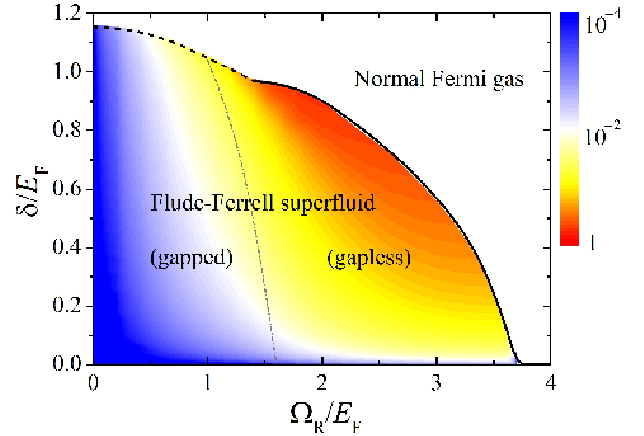


FIG. 1: (color online) Phase diagram of a 3D spin-orbit coupled atomic Fermi gas at a broad Feshbach resonance and at the lowest experimentally attainable temperature $0.05T_F$. The synthetic spin-orbit coupling is induced by two counter-propagating Raman laser beams with strength Ω_R and detuning δ . Here we take the recoil momentum $k_R = k_F$. A narrow LO wedge near $\Omega_R \sim 0$ and $\delta \sim 1.2E_F$ is not shown. By increasing δ , the Fermi cloud changes from a FF superfluid to a normal gas, via first-order (dashed line) and second-order (solid line) transitions at low and high Ω_R , respectively. The FF superfluid can be either gapped or gapless. These two phases are separated by the dot-dashed line. The color shows the magnitude of the centre-of-mass momentum of Cooper pairs for the FF superfluid, q/k_F . The BCS superfluid occurs at $\Omega_R = 0$ or $\delta = 0$ only.

unfortunately, the FFLO phase occupies only an extremely small volume in parameter space [14, 15] and thus is impossible to observe experimentally [16, 17].

In this Letter, we predict that the FF superfluid could be easily observed in a resonantly interacting 3D atomic

Fermi gas with synthetic spin-orbit coupling. This system was recently realized experimentally at Shanxi University [18] and at Massachusetts Institute of Technology [19], using a two-photon Raman process. Our main result is summarized in Fig. 1. At experimentally accessible temperatures ($T \sim 0.05T_F$) [17] the FF state occupies a major part of the phase diagram. By tuning the strength (Ω_R) and detuning (δ) of two Raman laser beams, the centre-of-mass momentum of Cooper pairs q can be as large as the Fermi momentum k_F . We propose that such a large FF momentum can be easily measured by momentum-resolved radio-frequency spectroscopy (see Fig. 4) [20]. We note that the emergence of a FF superfluid has also been predicted in a 3D atomic Fermi gas in the presence of 2D Rashba spin-orbit coupling [21] or 3D isotropic spin-orbit coupling [22], or in a two-dimensional atomic Fermi gas with a two-photon Raman process [23]. These systems are yet to be realized experimentally. We also note that the possibility of the FF superfluidity in our setting has been discussed very recently by Vijay Shenoy [24].

We start by considering a 3D spin-orbit coupled spin-1/2 Fermi gas of ^6Li or ^{40}K atoms near a broad Feshbach resonance [18, 19]. Experimentally, the synthetic spin-orbit coupling is induced using two counter-propagating Raman laser beams (i.e., along the z -direction) that couple the two different spin states, following the same scenario as in the NIST experiment for a ^{87}Rb Bose-Einstein condensate (BEC) [25]. The Raman process can be described by, $(\Omega_R/2) \int d\mathbf{x} [\Psi_\uparrow^\dagger(\mathbf{x}) e^{i2k_R z} \Psi_\downarrow(\mathbf{x}) + \text{H.c.}]$, where $\Psi_\sigma^\dagger(\mathbf{x})$ is the creation field operator for atoms in the spin-state σ , Ω_R is the coupling strength of Raman beams, and $k_R = 2\pi/\lambda_R$ is the recoil momentum determined by the wave length λ_R of the two beams. Thus, during the two-photon Raman process, a momentum of $2\hbar k_R$ is imparted to an atom while its spin is changed from $|\downarrow\rangle$ to $|\uparrow\rangle$. This creates a correlation between the spin and orbital motion, which can be seen more clearly by taking a gauge transformation, $\Psi_\uparrow(\mathbf{x}) = e^{ik_R z} \psi_\uparrow(\mathbf{x})$ and $\Psi_\downarrow(\mathbf{x}) = e^{-ik_R z} \psi_\downarrow(\mathbf{x})$. Near a Feshbach resonance, the system may therefore be described by a single-channel model Hamiltonian $\mathcal{H} = \int d\mathbf{x} [\mathcal{H}_0 + \mathcal{H}_{int}]$, where the single-particle part

$$\mathcal{H}_0 = \begin{bmatrix} \psi_\uparrow^\dagger & \psi_\downarrow^\dagger \end{bmatrix} \begin{bmatrix} \hat{\xi}_{\mathbf{k}} + \lambda \hat{k}_z + \delta/2 & \Omega_R/2 \\ \Omega_R/2 & \hat{\xi}_{\mathbf{k}} - \lambda \hat{k}_z - \delta/2 \end{bmatrix} \begin{bmatrix} \psi_\uparrow \\ \psi_\downarrow \end{bmatrix}, \quad (1)$$

and $\mathcal{H}_{int} = U_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow(\mathbf{x})$ is the interaction Hamiltonian that describes the contact interaction between two spin states with strength U_0 . The interaction strength should be expressed in terms of the s -wave scattering length a_s , i.e., $1/U_0 = m/(4\pi\hbar^2 a_s) - 1/V \sum_{\mathbf{k}} m/(\hbar^2 k^2)$, which can be tuned precisely by sweeping an external magnetic field around the Feshbach resonance [8]. Here V is the volume of the system. In the single-particle Hamiltonian (1), $\hat{k}_z \equiv -i\partial_z$, $\hat{\xi}_{\mathbf{k}} \equiv -\hbar^2 \nabla^2/(2m) - \mu$ af-

ter dropping a constant recoil energy, and δ is the two-photon detuning from the Raman resonance. For convenience, we have defined a spin-orbit coupling constant $\lambda \equiv \hbar^2 k_R/m$. In the Shanxi experiment with ^{40}K atoms [18], the Fermi wavelength is about $k_F \simeq 1.6k_R$ and the coupling strength $\Omega_R \simeq 1.5E_R \simeq 0.6E_F$, in units of the recoil energy $E_R \equiv \hbar^2 k_R^2/(2m)$ or Fermi energy $E_F \equiv \hbar^2 k_F^2/(2m)$.

For the model Hamiltonian (1), most previous theoretical studies focused on the case with $\delta = 0$ and considered the standard BCS superfluidity [26, 27]. In this work, we are interested in the FF pairing state with a single valued center-of-mass momentum. Our investigation is motivated by the interesting finding that the two-body bound state at $\delta \neq 0$ always acquires a finite center-of-mass momentum along the z -axis [28]. At the many-body level, we therefore anticipate that at finite detunings a FF superfluid could be more favorable than a BCS superfluid [24].

By assuming a FF-like order parameter $\Delta(\mathbf{x}) = -U_0 \langle \psi_\downarrow(\mathbf{x}) \psi_\uparrow(\mathbf{x}) \rangle = \Delta e^{iqz}$, we consider the mean-field decoupling of the interaction Hamiltonian, $\mathcal{H}_{int} \simeq -[\Delta(\mathbf{x}) \psi_\uparrow^\dagger(\mathbf{x}) \psi_\downarrow^\dagger(\mathbf{x}) + \text{H.c.}] - \Delta^2/U_0$. Then, using a Nambu spinor $\Phi(\mathbf{x}) \equiv [\psi_\uparrow, \psi_\downarrow, \psi_\uparrow^\dagger, \psi_\downarrow^\dagger]^T$, the total Hamiltonian can be written in a compact form, $\mathcal{H} = (1/2) \int d\mathbf{x} \Phi^\dagger(\mathbf{x}) \mathcal{H}_{BdG} \Phi(\mathbf{x}) - V \Delta^2/U_0 + \sum_{\mathbf{k}} \hat{\xi}_{\mathbf{k}}$, where the Bogoliubov Hamiltonian

$$\mathcal{H}_{BdG} \equiv \begin{bmatrix} \mathcal{S}_{\mathbf{k}}^+ & \Omega_R/2 & 0 & -\Delta(\mathbf{x}) \\ \Omega_R/2 & \mathcal{S}_{\mathbf{k}}^- & \Delta(\mathbf{x}) & 0 \\ 0 & \Delta^*(\mathbf{x}) & -\mathcal{S}_{-\mathbf{k}}^+ & -\Omega_R/2 \\ -\Delta^*(\mathbf{x}) & 0 & -\Omega_R/2 & -\mathcal{S}_{-\mathbf{k}}^- \end{bmatrix} \quad (2)$$

and $\mathcal{S}_{\mathbf{k}}^\pm \equiv \hat{\xi}_{\mathbf{k}} \pm \lambda \hat{k}_z \pm \delta/2$. It is straightforward to diagonalize the Bogoliubov Hamiltonian $\mathcal{H}_{BdG} \Phi_{\mathbf{k}\eta}(\mathbf{x}) = E_{\mathbf{k}\eta} \Phi_{\mathbf{k}\eta}(\mathbf{x})$ with quasiparticle wave-function $\Phi_{\mathbf{k}\eta}(\mathbf{x}) = 1/\sqrt{V} e^{i\mathbf{k}\cdot\mathbf{x}} [u_{\mathbf{k}\eta\uparrow} e^{+iqz/2}, u_{\mathbf{k}\eta\downarrow} e^{+iqz/2}, v_{\mathbf{k}\eta\uparrow} e^{-iqz/2}, v_{\mathbf{k}\eta\downarrow} e^{-iqz/2}]^T$ and quasiparticle energy $E_{\mathbf{k}\eta}$ ($\eta = 1, 2, 3, 4$). The mean-field thermodynamic potential Ω at temperature T is then given by

$$\frac{\Omega}{V} = \frac{1}{2V} \left[\sum_{\mathbf{k}} (\xi_{\mathbf{k}+\mathbf{q}/2} + \xi_{\mathbf{k}-\mathbf{q}/2}) - \sum_{\mathbf{k}\eta} E_{\mathbf{k}\eta} \right] - \frac{k_B T}{V} \sum_{\mathbf{k}\eta} \ln \left(1 + e^{-E_{\mathbf{k}\eta}/k_B T} \right) - \frac{\Delta^2}{U_0}. \quad (3)$$

Note that the summation over the quasiparticle energy must be restricted to $E_{\mathbf{k}\eta} \geq 0$ because of an inherent particle-hole symmetry in the Nambu spinor representation [29]. For a given set of parameters (i.e., the temperature T , interaction strength $1/k_F a_s$ etc.), different mean-field phases can be determined using the self-consistent stationary conditions: $\partial\Omega/\partial\Delta = 0$, $\partial\Omega/\partial q = 0$, as well as the conservation of total atom number, $N = -\partial\Omega/\partial\mu$. At finite temperatures, the ground state has the lowest free energy $F = \Omega + \mu N$.

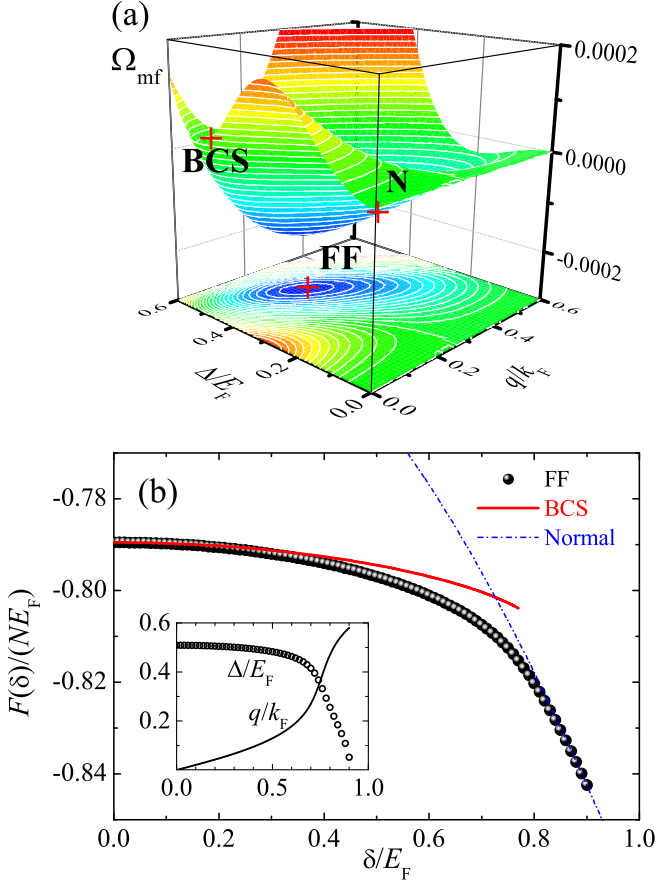


FIG. 2: (color online) (a) Landscape of the thermodynamic potential, $\Omega(\Delta, q) - \Omega(0, 0)$, at $\Omega_R = 2E_F$ and $\delta = 0.68E_F$. The chemical potential is fixed to $\mu = -0.471E_F$. The competing ground states include (i) a normal Fermi gas with $\Delta = 0$; (ii) a fully paired BCS superfluid with $\Delta \neq 0$ and $q = 0$; and (iii) a finite momentum paired FF superfluid with $\Delta \neq 0$ and $q \neq 0$. (b) The free energy of different competing states as a function of the detuning at $\Omega_R = 2E_F$. The inset shows the detuning dependence of the order parameter and momentum of the FF superfluid state.

Without loss of generality, hereafter we consider the resonance case with a divergent scattering length $1/k_F a_s = 0$ and set $T = 0.05T_F$, which is the lowest temperature that has been reported for Fermi atoms [17]. According to the typical number of atoms in experiments [18, 19], we shall take $k_R = k_F$, corresponding to a dimensionless spin-orbit coupling constant $\lambda k_F/E_F = 2$. The Raman coupling strength Ω_R and detuning δ can be easily tuned experimentally. Thus, we focus on the phase diagram as functions of Ω_R and δ .

In general, for any set of parameters there are three competing ground states that are stable against phase separation (i.e., $\partial^2\Omega/\partial\Delta^2 \geq 0$), as shown in Fig. 2(a): normal gas ($\Delta = 0$), BCS superfluid ($\Delta \neq 0$ and $q = 0$), and FF superfluid ($\Delta \neq 0$ and $q \neq 0$). Remarkably, in the presence of spin-orbit coupling (i.e., $\Omega_R \neq 0$) the

FF superfluid is always more favorable in energy than the standard BCS pairing state at finite detunings (Fig. 2(b)). It is easy to check that the superfluid density of the BCS pairing state in the axial direction becomes negative (i.e., $\partial\Omega/\partial q < 0$), signaling the instability towards a FF superfluid. Therefore, experimentally the Fermi cloud would always condense into a FF superfluid at finite Raman detunings. In Fig. 1, we report a low-temperature phase diagram that could be directly observed in current experiments. The FF superfluid occupies the major part of the phase diagram.

The greatly enlarged parameter space for a FF superfluid can be qualitatively understood from the change of the Fermi surface due to spin-orbit coupling, as discussed in the pioneering work by Barzykin and Gor'kov [30]. In our case, the single-particle energy spectrum takes the form, $E_{\mathbf{k}\pm} = \hbar^2 k^2/2m \pm \sqrt{(\Omega_R/2)^2 + (\lambda k_z + \delta/2)^2}$, where “ \pm ” stand for two helicity bands. At large spin-orbit coupling ($\Omega_R \gg \lambda k_F, \delta$), we find

$$E_{\mathbf{k}\pm} \simeq \frac{\hbar^2 (k_x^2 + k_y^2)}{2m} + \frac{\hbar^2}{2m} \left(k_z \pm \frac{q}{2} \right) \pm \frac{\Omega_R}{2}. \quad (4)$$

where $q = 2k_R\delta/\Omega_R$. Thus, the Fermi surfaces remain approximately circular but the centers of the two Fermi surfaces are shifted by $q/2$ along the z -axis in opposite directions. In this situation, we may gain condensate energy by shifting the two Fermi surfaces by an amount $\pm(q/2)\mathbf{e}_z$, respectively [31]. Here \mathbf{e}_z is the unit vector in z -axis. The fermionic pairing then occurs between single-particle states of $\mathbf{k} + (q/2)\mathbf{e}_z$ and $-\mathbf{k} + (q/2)\mathbf{e}_z$, leading to a FF order parameter that has a spatial distribution $\Delta(\mathbf{x}) = \Delta e^{iqz}$. The direction of the FF momentum is uniquely fixed by the form of spin-orbit coupling (i.e., λk_z) and its magnitude is roughly proportional to the detuning δ .

At small spin-orbit coupling $\Omega_R \sim 0$, however, the above argument becomes invalid. Indeed, in the absence of spin-orbit coupling ($\Omega_R = 0$) the instability to inhomogeneous superfluidity is driven solely by the detuning. The FF superfluid occurs in a narrow window (not shown in Fig. 1): $\delta_{cs} \leq \delta \leq 1.204E_F$, where $\delta_{cs} \simeq 1.162E_F$ is the Clogston limit for a unitary Fermi gas [15]. The direction of the FF momentum cannot be specified, indicating that a LO state with periodic stripe structure is more preferable [32]. In our calculations, we do not consider such a LO superfluid that exists in small wedge near $\Omega_R \sim 0$ and $\delta \sim 1.2E_F$. Due to this simplification, at low Raman coupling strengths the FF superfluid does not transform continuously into a normal gas.

The FF superfluid in Fig. 1 can have different topological structures in the quasiparticle excitation spectrum, as illustrated in Fig. 3. At small spin-orbit couplings (Fig. 3(a)), the FF superfluid has a negligible center-of-mass momentum and therefore behaves very similar to a standard gapped BCS superfluid, while at large spin-orbit coupling (Fig. 3(b)),

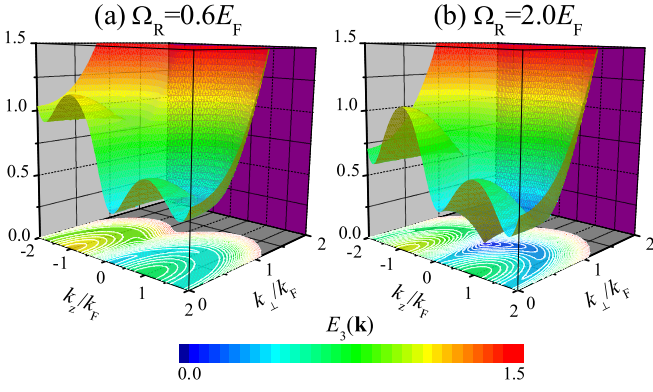


FIG. 3: (color online) The excitation spectrum $E_3(\mathbf{k})$ (in units of E_F) as functions of $k_\perp = \sqrt{k_x^2 + k_y^2}$ and k_z : (a) gapped spectrum at $\Omega_R = 0.6E_F$ and (b) gapless spectrum at $\Omega_R = 2E_F$. Here we take $\delta = 0.5E_F$. Due to the particle-hole symmetry, $E_2(\mathbf{k}) = -E_3(-\mathbf{k})$ and $E_1(\mathbf{k}) = -E_4(-\mathbf{k})$. The gapped branches $E_1(\mathbf{k})$ and $E_4(\mathbf{k})$ are not shown.

the sizable FF momentum features a gapless spectrum. The nodal points with zero-excitation energy are located at or close to the $k_z = 0$ axis, along which the dispersion relations take the form, $E(k_\perp, 0) = \pm \sqrt{(\lambda q + \delta)^2 + \Omega_R^2/2} \pm \sqrt{\Delta^2 + (\hbar^2 k_\perp^2/2m + \hbar^2 q^2/8m)^2}$. Apparently, these nodal points would appear when the effective Zeeman field $\sqrt{\delta^2 + \Omega_R^2/2}$ is sufficiently large. In Fig. 1, the gapped and gapless FF superfluids are separated by a dot-dashed line. The topological transition from gapped to gapless phases is continuous and could be revealed by thermodynamic measurements through the atomic compressibility, spin susceptibility and momentum distribution.

We now consider how to experimentally detect a FF superfluid through momentum-resolved rf-spectroscopy [20], which is a cold-atom analog of the widely used angle-resolved photoemission spectroscopy in solid-state systems. In such a measurement, one uses a rf field with frequency ω to break a Cooper pair and transfer the spin-down atom to an un-occupied third hyperfine state $|3\rangle$. The momentum distribution of the transferred atoms is then measured by absorption imaging after a time-of-flight expansion. The rf Hamiltonian can be written as $\mathcal{H}_{rf} \propto \int d\mathbf{x} [e^{-ik_R z} \psi_3^\dagger(\mathbf{x}) \Psi_\downarrow(\mathbf{x}) + \text{H.c.}]$, where $\psi_3^\dagger(\mathbf{x})$ is the field operator which creates an atom in $|3\rangle$. The momentum transfer $k_R \mathbf{e}_z$ in \mathcal{H}_{rf} arises from the gauge transformation. For a weak rf drive, the number of transferred atoms can be calculated using linear response theory

$$\Gamma(\mathbf{k}, \omega) = \mathcal{A}_{\downarrow\downarrow} \left[\mathbf{k} + (k_R - \frac{q}{2})\mathbf{e}_z, \xi_{\mathbf{k}} - \omega \right] f(\xi_{\mathbf{k}} - \omega). \quad (5)$$

Here $\mathcal{A}_{\downarrow\downarrow}(\mathbf{k}, \omega) = \sum_\eta |u_{\mathbf{k}\eta\downarrow}|^2 \delta(\omega - E_{\mathbf{k}\eta})$ is the single-particle spectral function of spin-down atoms, $\xi_{\mathbf{k}} \equiv \hbar^2 k^2/(2m) - \mu$, and $f(x) \equiv 1/(e^{x/k_B T} + 1)$ is the Fermi-Dirac distribution function. For a FF superfluid, as Cooper pairs now carry a finite center-of-mass momen-

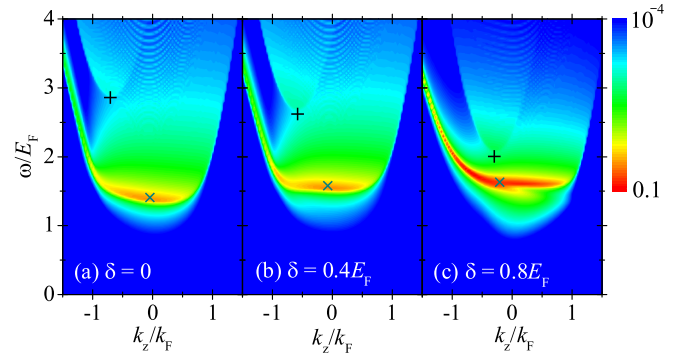


FIG. 4: (color online) Logarithmic contour plot of momentum-resolved rf spectroscopy: number of transferred atoms $\Gamma(k_z, \omega)$ at $\Omega_R = 2E_F$ and at three detunings: (a) $\delta = 0$ and $q = 0$, (b) $\delta = 0.4E_F$ and $q \simeq 0.1k_F$, and (c) $\delta = 0.8E_F$ and $q \simeq 0.6k_F$.

tum $\mathbf{q} = q\mathbf{e}_z$, the transferred atoms acquire an overall momentum $\mathbf{q}/2$. As a result, there is a $\mathbf{q}/2$ shift in the transferred strength Eq. (5), which in principle could be experimentally measured.

In Fig. 4, we report the momentum-resolved rf spectroscopy along the z -direction $\Gamma(k_z, \omega) \equiv \sum_{\mathbf{k}_\perp} \Gamma(\mathbf{k}, \omega)$ at $\Omega_R = 2E_F$ on a logarithmic scale. Quite generally, there are two contributions to the spectroscopy, corresponding to two different final states: after a Cooper pair is broken by a rf-photon, the remaining spin-up atoms can be in different helicity bands [33]. These two contributions are well separated in the frequency domain, with peak positions indicated by the symbols “+” and “x”, respectively. Interestingly, at finite detuning with a sizable FF momentum \mathbf{q} , the peak positions of the two contributions are shifted roughly in opposite directions by an amount $\mathbf{q}/2$. This provides clear evidence for observing the FF superfluid.

In summary, we have shown that the condensate state of a spin-orbit coupled Fermi gas is the long-sought inhomogeneous Fulde-Ferrell superfluid. Our prediction can be readily examined in current experiments, where a three-dimensional spin-orbit coupled atomic Fermi gas is created using a two-photon Raman process [18, 19]. The finite center-of-mass momentum carried by the inhomogeneous superfluid can be unambiguously measured by momentum-resolved radio-frequency spectroscopy. Our work complements relevant studies of solid-state systems, in which inhomogeneous superfluidity was predicted to be enhanced by Rashba spin-orbit interaction [30], but was difficult to confirm experimentally. The controllable setting of a spin-orbit coupled atomic Fermi gas opens a new direction to explore the fascinating inhomogeneous superfluidity.

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* Electronic address: xiajiliu@swin.edu.au

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